

Chapter 2 Solutions for Introduction to Robotics

1. a) Use (2.3) to obtain

$${}^A_B R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

- b) Use (2.74) to get

$$\alpha = 90 \text{ degrees}$$

$$\beta = 90 \text{ degrees}$$

$$\gamma = -90 \text{ degrees}$$

2. a) Use (2.64) to obtain

$${}^A_B R = \begin{bmatrix} .330 & -.770 & .547 \\ .908 & .418 & .0396 \\ -.259 & .483 & .837 \end{bmatrix}$$

- b) Answer is the same as in (a) according to (2.71)

3. Use (2.19) to obtain the transformation matrices. The rotation is X-Y-Z fixed angles, so use (2.64) for that 3×3 submatrix, with angles

$$\gamma = 0 \text{ degrees}$$

$$\beta = -\sin^{-1} \left(\frac{\text{tripod_height}}{\text{distance_along_optical_axis}} \right) = -\sin^{-1} \left(\frac{1.5}{5} \right) = -107 \text{ degrees}$$

$$\alpha_C = 0 \text{ degrees}$$

$$\alpha_D = 120 \text{ degrees}$$

$$\alpha_E = 240 \text{ degrees}$$

The position vectors to the camera-frame origins are

$$\begin{aligned}
{}^B P_{CORG} &= \begin{bmatrix} \text{horizontal_distance} \\ 0 \\ \text{tripod_height} \end{bmatrix} = \begin{bmatrix} 4.77 \\ 0 \\ 1.50 \end{bmatrix} \\
{}^B P_{DORG} &= \begin{bmatrix} \text{horizontal_distance} \times \cos \alpha_D \\ \text{horizontal_distance} \times \sin \alpha_D \\ \text{tripod_height} \end{bmatrix} = \begin{bmatrix} -2.39 \\ 4.13 \\ 1.5 \end{bmatrix} \\
{}^B P_{EORG} &= \begin{bmatrix} \text{horizontal_distance} \times \cos \alpha_E \\ \text{horizontal_distance} \times \sin \alpha_E \\ \text{tripod_height} \end{bmatrix} = \begin{bmatrix} -2.38 \\ -4.13 \\ 1.50 \end{bmatrix},
\end{aligned}$$

where $\text{horizontal_distance} = \sqrt{(\text{distance_along_optical_axis})^2 - (\text{tripod_height})^2}$.

Combining the rotation and translation yields the transformation matrices via (2.19) as

$$\begin{aligned}
{}^B T_C &= \begin{bmatrix} -.300 & 0 & -.954 & 4.77 \\ 0 & 1.00 & 0 & 0 \\ .954 & 0 & -.300 & 1.50 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
{}^B T_D &= \begin{bmatrix} .150 & -.866 & .477 & -2.39 \\ -.260 & -.500 & -.826 & 4.13 \\ .954 & 0 & -.300 & 1.50 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
{}^B T_E &= \begin{bmatrix} .150 & .866 & .477 & -2.39 \\ .260 & -.500 & .826 & -4.13 \\ .954 & 0 & -.300 & 1.50 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}$$

4. The camera-frame origin is located at ${}^B P_{CORG} = [7 \ -2 \ 5]^T$. Use (2.19) to get the transformation, ${}^B T_C$. The rotation is Z-Y-X Euler angles, so use (2.71) with

$$\alpha = 0 \text{ degrees}$$

$$\beta = -110 \text{ degrees}$$

$$\gamma = -20 \text{ degrees}$$

to get

$${}^B T_C = \begin{bmatrix} -.342 & .321 & -.883 & 7.00 \\ 0 & .940 & .342 & -2.00 \\ .940 & .117 & -.321 & 5.00 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. Let

$${}^B P_1 = {}^B P_0 + 5 {}^B V_0 = [9.5 \quad 1.00 \quad -1.50]^T$$

The object's position in $\{A\}$ is

$${}^A P_1 = {}^A_B T {}^B P_1 = [-4.89 \quad 2.11 \quad 3.60]^T$$

6. (2.1)

$$\begin{aligned} R &= \text{rot}(\hat{Y}, \phi) \text{rot}(\hat{Z}, \theta) \\ &= \begin{bmatrix} c\phi & 0 & s\phi \\ 0 & 1 & 0 \\ -s\phi & 0 & c\phi \end{bmatrix} \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} c\phi c\theta & -c\phi s\theta & s\phi \\ s\theta & c\theta & 0 \\ -s\phi c\theta & s\phi s\theta & c\phi \end{bmatrix} \end{aligned}$$

7. (2.2)

$$\begin{aligned} R &= \text{rot}(\hat{X}, 60) \text{rot}(\hat{Y}, -45) \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & .500 & -.866 \\ 0 & .866 & .500 \end{bmatrix} \begin{bmatrix} .707 & 0 & -.707 \\ 0 & 1 & 0 \\ .707 & 0 & .707 \end{bmatrix} \\ &= \begin{bmatrix} .707 & 0 & -.707 \\ -.612 & .500 & -.612 \\ .353 & .866 & .353 \end{bmatrix} \end{aligned}$$

8. (2.12) Velocity is a “free vector” and only will be affected by rotation, and not by translation:

$$\begin{aligned} {}^A V &= {}^A_B R {}^B V = \begin{bmatrix} .707 & 0 & -.707 \\ -.612 & .500 & -.612 \\ .353 & .866 & .353 \end{bmatrix} \begin{bmatrix} 30.0 \\ 40.0 \\ 50.0 \end{bmatrix} \\ &= [-14.1 \quad -29.0 \quad 62.9]^T \end{aligned}$$

9. (2.31)

$${}^C_B T = \begin{bmatrix} 0 & 0 & -1 & 2 \\ .500 & -.866 & 0 & 0 \\ -.866 & -.500 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

10. (2.37) Using (2.45) we get that

$${}^B P_{AORG} = -{}^A_B R^T {}^A P_{AORG} = - \begin{bmatrix} .25 & .87 & .43 \\ .43 & -.50 & .75 \\ .86 & .00 & -.50 \end{bmatrix} \begin{bmatrix} 5.0 \\ -4.0 \\ 3.0 \end{bmatrix} = \begin{bmatrix} .94 \\ -6.4 \\ -2.8 \end{bmatrix}$$